The Mathematics of the Monarchs: Enriching the Butterfly Project

Brian Sharp, Indiana University of Pennsylvania <bds@iup.edu>

Abstract

The raising, tagging and releasing of Monarch butterflies is a staple in many science classes. However, there are several aspects of Monarch butterflies that can be used as a basis for interesting mathematics lessons. In addition to providing examples of activities related to Monarch butterflies, this article also discusses ways in which students might think about the mathematical concepts involved.

Ask any student at Eisenhower Elementary to name their favorite science unit and chances are that they will say "The Butterfly Unit." Each year teachers, like Mrs. Kaz and others, have their students raise Monarch butterflies that they tag and release into the wild as part of the *Monarch Watch Project*, an International Monarch butterfly tracking program based at the University of Kansas. As part of her science unit, Mrs. Kaz's students get to make first-hand observations of how a caterpillar becomes a butterfly. Her students can describe the various stages of a butterfly's life cycle, identify the parts of a butterfly's body, and talk about the migration patterns of Monarch butterflies. They can even instruct you on how to properly tag a butterfly so that researchers can collect data about the migration patterns of the Monarchs.

After having two sons enthusiastically participate in this project, I began to explore ways in which the science objectives in the project could be enriched with mathematical objectives to create a multidisciplinary unit that supports the Science, Technology, Engineering, and Mathematics (STEM) philosophy and adheres to both the National Council of Teachers of Mathematics (NCTM) curricular focal points for middle school mathematics and the Common Core State Standards for Mathematics CCSSM). My endeavors were given a boost when NOVA(PBS) aired a special called "The Incredible Journey of the Butterflies," which can be viewed at http://www.pbs.org/wgbh/nova/butterflies. The show describes in stunning detail and imagery how, over the span of several generations, Monarch butterflies leave the rainforests near Michoacán, Mexico, travel to North America, and then return to the same rainforests in Mexico. From this show, I extracted a few facts which I turned into mathematical activities.

Monarchs on the Move

The CCSSM Ratios and Proportional Relationships Standards for Grade 7 states that students should "analyze proportional relationships and use them to solve real-world and mathematical problems." The flight patterns of Monarchs serve as a basis for an interesting activity involving ratios and rates. According to the show, scientists have determined that a Monarch butterfly that begins the migration back to Michoacán, Mexico (from Canada) has to fly approximately 50 miles a day to complete its journey before the cold weather arrives. Armed with this bit of knowledge, a scientist who observes a large band of butterflies on a given day can estimate the location of these butterflies in subsequent days. Similar to work that scientists may do to estimate butterfly locations, students participating in the *Monarchs on the Move* estimate the location of the butterflies that they release from their school after various days of flight. This is generally a very motivating activity because students are naturally curious as to how far a butterfly can travel in one day from their respective schools.

Monarch Butterfly Fact

The butterflies that return to Mexico each fall are the great-grandchildren of the butterflies that left Mexico at the end of the previous winter.

To conduct this investigation, teachers will need to acquire a map of their area that is appropriate in scale. The use of maps relate to part of the NCTM Measurement and Geometry Focal Point for Grade 7 which states students should "solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects." Figure 1 shows a map of the area around Washington, D.C. obtained from Google Maps. Notice that the map was re-sized such that the 50-mile interval was approximately ½-inch long.

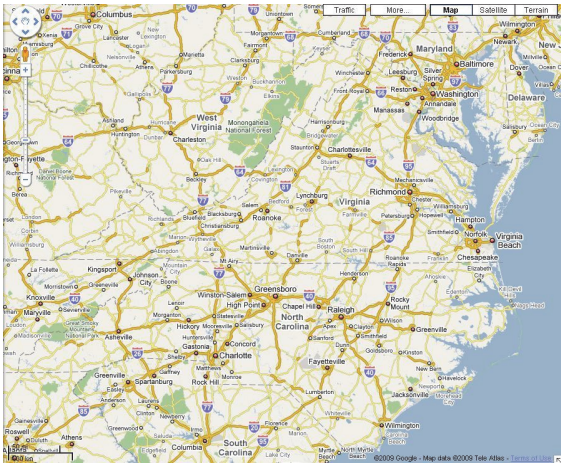

Figure 1. A map of Washington, D.C. (Google Maps)

After each student has a map, the teacher can ask students to make a conjecture as to where a butterfly released from the school on a Monday morning would be on Tuesday morning. Once the conjectures have been formulated and discussed, the teacher can lead the students through the process of constructing a circle with a 50-mile radius centered at their school using compasses, rulers, and the scale of the map as measurement tools. The students should be able to verbalize that the radius of the circle represents the approximate distance that a butterfly might travel in a day. As an extension to the activity, teachers can ask their students questions like "Which town could a butterfly reach in two days?" or "How many days will it take a butterfly to travel to Town B?" Figure 2 shows concentric circles around Washington, D.C. that correspond to the distance a butterfly could fly in four days.

Figure 2. A map of Washington, D.C. showing circles of various radii

Adding a Sense of Direction

It is interesting to note that scientists have found that butterflies somehow know what azimuth (e.g., heading on a map) to fly to lead them almost directly towards Michoacán, Mexico. Even butterflies that are moved from their birthplace to a new location before they are released quickly adjust their flight patterns so that they don't fly to the wrong location. For example, from Washington, D.C., the butterflies generally fly in a southwesterly direction towards Michoacán using an approximate azimuth of S38°W. [The notation S38°W means that the direction of flight from Washington D.C. is 38° west of due south.]

Monarch Butterfly Fact
The Monarch butterflies that leave Canada fly about 2,000 miles in two months to reach Mexico.

In order for students to determine an approximate azimuth from their school to Michoacán, Mexico, they need a map that includes both locations. Using the location of their school as the vertex of an angle, due south as the initial side of the angle, and the ray connecting their school to Michoacán as the terminal side of the angle, students can use a protractor to calculate the azimuth that Monarchs might use to fly to their winter breeding grounds. Figure 3 shows the setup for determining the azimuth from Washington, D.C. to Michoacán, Mexico.

Figure 3. Determining the azimuth from Washington, D.C. to Michoacán, Mexico

Thinking about Students' Thinking

It is often interesting to see the many strategies that students create to solve a problem like "Where could a butterfly be located after three days of flight?" Since the scale on the map is fixed, students have to come up with some strategy to determine the length of the interval that corresponds to two days of flight or three days of flight. Several solution methods are possible and each method that students use provides the teacher with insights into their students' abilities and thought processes.

One such method is the "Copy and Pasting" method. Students using this method, mark the distance that a butterfly could fly in one day on their ruler. To determine the distance that a butterfly could fly in two days, they just copy the interval onto their map twice in a linear fashion. Little or no calculations are needed for this method, except for the initial determination of the length of one day's flight. A second method is the 'Repeated Addition" method. Students using this method will determine the actual length on their ruler that corresponds to the distance that a butterfly could fly in one day, say one half, or 0.5, inch. To determine the distance that a butterfly could fly in two days, they add 0.5 + 0.5 to get a sum of 1 inch. They then mark off a 1 inch interval on the map as the radius of the "2 day circle." A third method is the "Multiplicative" method. This method is similar to the Repeated Addition method, except that students use multiplication facts instead of addition facts (e.g., 2 × 0.5 inch = 1 inch).

Origins and the Origin

The "Methuselah" generation of Monarch butterflies is the generation of butterflies that are born in various locations of North America near the end of summer. These butterflies have to fly all the way back to the Mexican rainforests. They are called the "Methuselah" generation because they lived up to nine times as long as other Monarch butterflies. Methuselah was a biblical figure who, according to the Hebrew Bible, lived for 969 years.

Many schools that raise and release "Methuselah" Monarchs tag the butterflies with special stickers that do not hurt the butterflies or hinder their ability to fly. These stickers contain a code that can be linked to information such as when and where the butterflies were released. Researchers try to collect the tagged butterflies and analyze the information on the stickers to gain insights into the Monarchs' migration patterns. Pictures of tagged butterflies can be viewed on the Monarch Watch website at: http://monarchwatch.org/gallery/photo/tag1.htm.

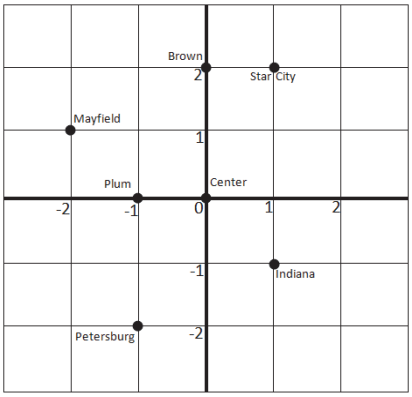

Figure 4. A Fictitious Map using the Cartesian Coordinate System

Monarch Butterfly Fact
100-million Methuselah butterflies start the journey towards Mexico each year.

In the *Origins and the Origin* activity, students try to analyze the information from a group of "tagged" butterflies to determine from where the butterflies were released. To do this activity, teachers will need a collection of toy (or paper) butterflies, small stickers, tape or magnets, and a "map" that can be projected onto the blackboard. The fictional map, shown in Figure 4, uses the Cartesian coordinate system. Maps can be adapted to use longitude and latitude, like a world map, or numbers and letters, like a state map.

After the map is created, teachers divide the class into two groups, the "Taggers" and the "Catchers." The Taggers are given the toy butterflies, stickers, and some markers. Each Tagger is assigned a town from which they will "release" their respective butterfly. The Tagger writes the coordinates of their assigned town on their sticker and places the sticker onto their butterfly. For example, if a Tagger is assigned Indiana as their release location, the Tagger would write the coordinates (1, -1) on their sticker before they place the sticker on their butterfly. Once the butterflies are tagged, the Taggers hide their butterflies around the room while the Catchers cover their eyes. Next, each Catcher tries to "capture" one of the hidden butterflies. When a Catcher captures a butterfly, they use the map and the information on the sticker to determine the town from which the butterfly was released and mark the location on the map.

When doing this type of activity, teachers may notice that some students can easily determine a point on a grid given an ordered pair, but have difficulty doing the reverse (e.g., identifying the ordered pair related to a plotted point). Other students can identify the ordered pair of a plotted point, but may have difficulty plotting a point given an ordered pair. Hence, the students should trade roles as Taggers and Catchers and repeat the activity multiple times. The repetition of the activity allows students the opportunity to gain a better understanding of the Cartesian coordinate system.

Conclusion

Monarch butterflies are fascinating creatures that students love to study. However, most of the students' experiences with the Monarchs have come in science classes. With the activities described in this article, teachers can use some of the excitement generated by the study of Monarch butterflies to motive the learning of mathematical concepts.

Monarch Butterfly Fact
The Mazahua people in Mexico believe that the returning Monarchs represent the spirits of their ancestors. They make masks to honor their ancestors.

References

Nick de Pencier, Director, *The Incredible Journey of the Butterflies*. (Ontario, Canada: Films à Trois, 2009). Available at: http://www.pbs.org/wgbh/nova/butterflies.

The Monarch Watch, sponsored by the University of Kansas, website: http://www.monarchwatch.org.

Brian Sharp is an Associate Professor of Mathematics at Indiana University of Pennsylvania. His major area of interest includes using digital pictures and videos to enhance K-12 mathematics instruction. He is currently trying to develop innovative instructional modules that combine the STEM-related fields into a coherent curriculum.

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- Wiki: http://cincymathcircle.wikispaces.com/
- Facebook: https://www.facebook.com/cincymathcircle
- Started in 2011 with over 100 participants each year with six upcoming sessions this school year, including the upcoming OCTM Conference [Session 55: Liar's Bingo! Thursday 12:00 - 1:20]
- Supported by the University of Cincinnati Department of Mathematics MAT Program, Math for America seed grant, and Madeira High School (facilities)

Southeast Ohio Math Teachers' Circle

- Website: www.seomtc.weebly.com Email: seomtcircle@gmail.com
- Bi-monthly meetings throughout the 2014-2015 school year: September, November, January, March, and May. Typically held Saturdays 9 am- noon in the Athens area.
- Funded by Ohio Board of Regents *Improving Teacher Quality* grant

Columbus Math Teachers' Circle

- Website: http://columbusmathcircle.wordpress.com
- Monthly meetings throughout 2014-2015 school year: 2nd Tuesday of every month starting October 14, 2014 from 5:30 pm - 8 pm at Hastings Middle School in Upper Arlington.
- Funded by grants from American Institute of Mathematics (AIM) and OSU's Steam Factory.